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Tuning Systems for Keyboard Instruments – A Historical Overview

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TEMPERAMENT AND TEMPERING

Introduction

rom the multitude of existing or documented tuning systems, I chose to focus on European space and its richest period in this respect, 17th- and 18th-century music. Baroque temperaments came back with the early music movement of rediscovering and revalorizing ancient music by turning to historical sources and period instruments. As such, I based my presentation and reconstruction of the analysed tuning systems on the historical source. The imprecision of some of the descriptions engendered several possible interpretations and, subsequently, interesting comparisons. The interpretation of the sources was always strictly personal and therefore original. I supplemented mathematical calculations with diagrams which very suggestively illustrate a particular tuning system or the differences between various temperaments. On account of the experience I gained over the years as harpsichord and organ tuner, I took the liberty to complement these analyses with certain subjective aspects as to the perception of sonorities in different temperaments.

To measure, determine, and compare interval values and the deviation from pure intervals I chose the cent, as it allows for a much more efficient additive approach than the ratio. I didn't insist on the phenomenon of beat frequency, one of the tuner's most important instruments, because it would have exceeded the scope of this paper and the presentation would have become overloaded with information. The fifth and the major third are decisive in establishing the particularities of a certain temperament. As appropriate, I approached other intervals as well as chromatic tones.

Tempering

Tempering is the attempt to reach a compromise between the sound's natural attributes and the "artificial", chromatic 12-tone octave system established by European music. There is an incompatibility between the pure (natural) intervals and the composition of this artificial scale. The Pythagorean system of musical ratios which make up the Harmony of the Spheres became a lost paradise for European music, led into temptation as it was by modulation. What followed was a gradual de-hierarchisation of the 12-tone system to its dodecaphonic self-destruction. Had it not taken this course, the astonishing history of tonal European music would of course have been unthinkable. This incompatibility is apparent from the fact that musical ratios can't generate a non-hierarchical, non-preferential system. Not the ratios, but the system (laid on a Procrustean bed) is deficient!

A concatenation of twelve pure fifths starting for example from C, will lead us to B[#], higher than its enharmonic C by a Pythagorean comma (PC):

 $-(3/2)^{12} = 129,7463$ compared to $2^7 = 128$ thus PC = 1,013643 = 23,46 cents. Four successive pure fifths will result in a major third plus two octaves. The third thus obtained will be sensibly larger than the pure third, by a Syntonic comma (SC):

 $-(3/2)^4/2^2 = 81/64$ compared to 5/4 thus SC = 81/80 = 1,0125 = 21,5 cents. And three pure major thirds compose an augmented seventh differing by a Lesser diesis (LD) from the enharmonic octave:

 $-(5/4)^3 = 1,953125$ compared to 2 thus LD = 1,024 = 41,0589 cents.

These simple calculations show how the three fundamental intervals are interdependent within the system. As the octave is the only interval whose purity must be preserved, the fifth and the major third remain subject to tempering.

A very fitting and up-to-date synonym for tempering is *negotiation*. Indeed, by tempering we negotiate between the fifth's and the third's drive toward purity. The fifths' purity will mean sacrificing the thirds, and the thirds' purity implies forsaking the fifths' purity. We can also note that despite sacrificing the other interval it will be impossible to obtain only pure thirds or fifths.

This negotiation is conducted according to the targeted musical and aesthetic goals. Quite a number of tempering systems were used over time, aiming to arrive as close to the aesthetic ideal of a particular music as possible. The preference for either the fifth or the third (at the expense of the other interval) generated qualitatively different tuning systems which didn't evolve from, but replaced, one another in order to serve the musical requirements of their time or cultural space.

To support my endeavour to define tempering, I will call two famous figures in music, from two very different eras: Leopold Mozart and Paul Hindemith.

Leopold Mozart associates tempering with the practice of enharmonic equivalency and believe it to be, in the mid-18th century, imposed by reasons having to do with the 12-key keyboard.

> On the keyboard, G[#] and Ab, Db and C[#], F[#] and Gb, and so on, are one and the same note. This is caused by the temperament. But according to the right ratio, all notes lowered by a (b) are a comma higher than those raised by a (#). For example: Db is higher than C[#], Ab higher than the G[#], Gb than F[#] and so on. Here the good ear must judge, and it would indeed be well to introduce the pupils to the monochord.¹ (Mozart 1985: 70)

Interestingly, in his time *clean* violin intonation equates sharps with down and flats with up; it was only in the 19th century that intonation will obey the attraction of the leading note, reversing the rule. With ensembles specialising in Baroque music, such just intonation, the opposite of what we are used to, at first shocks just as much, or even more, than the sound of period instruments. It's worth noticing that Leopold Mozart urges his disciples to check their intonation in an extremely objective manner – against the monochord. This reference to a highly objective technical means makes it clear that in an era today often labelled as empirical much rigour was in fact demanded.

Hindemith employs an extremely suggestive allegory to explain temperament (irregular temperament), namely, the degree of a room's cleanliness and how the room is cleaned. Irregular temperament would be like "a lazy housemaid [deliberating] under which edge of the big carpet to put the

¹ "Auf dem Clavier sind Gis und As, Des und Cis, Fis und Ges u. s. f. eins. Das machet die Temperatur. Nach dem richtigen Verhältnisse aber sind alle die durch das (b) erniedrigten Töne um ein Komma höher als die durch das (\$) erhöheten Noten. Z. E. Des ist höher als Cis; As höher als Gis, Ges höher als Fis, u. s. w. Hier muss das gute Gehör Richter seyn: Und es wäre freilich gut, wenn man die Lehrlinge zu dem Klangmässer (Monochordon) führete" (Mozart 1756: 66).

sweepings" (Hindemith 1961: 99).² Hindemith prefers this particular manner over the alternative, of "distributing the total amount of dust all over the place so thinly that it cannot be seen very clearly, yet covers everything with a perceptible film"³ (Hindemith 1961: 103), which would correspond to equal temperament. Hindemith accepts equal temperament for keyboard instruments for the same practical reasons – we are of course in the heart of the 20th century – but he also says that pure intervals as intoned by quartets or vocal ensembles will sound better than when played by keyboard instruments, with the latter's slight but permanent out-of-tune-ness (deviation from purity, *Unreinheit*) (Hindemith 1961: 102).

More recent studies show an increased precision in defining temperament. Thus, starting from the two principles of intonational space, the harmonic ratios and the logarithmic scale of sound perception, Barlow distinguishes between two notions (Barlow 1980, 1987): tempering – the path from consonance to equidistance, and rationalisation – the opposite direction, from equidistance on the logarithmic scale to the purity of the intervals (see Fig. 1).



Fig. 1. Tempering / Rationalization.

This distinction, while precise, is not historically relevant – indeed, it would be so if we placed equal temperament in the middle, flanked by the Pythagorean and the meantone tuning systems. Shifting for centuries between these two criteria, musicians would eventually meet halfway and accept equal tempera-

² "[D]ie Überlegung einer faulen Reinemachefrau, unter welcher Ecke eines Teppichs sie den zusammengekehrten Schmutz verstecken soll" (Hindemith 1959: 114).

³ "Schließlich lieben wir, unser Wohnzimmer in sauberem Zustand zu haben, auch wenn sich in manchen Ecken unvermeidlicherweise etwas Staub ansetzt. Deshalb sind die Praktiken der kehrichtverbergenden Reinemachefrau ebenso unerwünscht wie die raffinierte Arbeit eines Gewitzten, der den Gesamtvorrat an Staub über die ganze Zimmer und für immer so dünn verteilt, daß er nicht völlig klar gesehen werden kann, aber alles mit einem doch noch wahrnehmbaren grauen Film überdeckt" (Hindemith 1959: 118).

ment. Barlow's analysis is significant because it brings back the notion of consonance, starting from Euler's studies which tried to establish mathematically the degree of consonance (gradus suavitatis) of an interval (the contents of prime numbers, the minimum exponent, for instance 5/4 v. 81/64). He creates a mathematical formula to set the values of harmonic consonance which he calls harmonicity and which pays attention to the indigestibility of whole numbers.

Equal temperament

Equal temperament is today's most known and frequently used keyboard instrument tuning system. Although irrelevant for the time period with which this paper is concerned, I will nevertheless begin by it because for today's musician, born and raised with and in it, it can represent a landmark and serve as comparison (but not as standard!). Ideally, historical tunings should be looked at chronologically, from the Middle Ages to Classicism and as change instead of evolution, not from the 21st century back towards the past and as a great preparatory stroke to our time. It must be stated clearly from the beginning that equal temperament was not an ideal to which irregular temperaments aspired over time, but a compromise reached by gradually abandoning the natural attributes of music: non-enharmonic equivalents, the intervals' drive toward purity, tonality hierarchisation etc. Equal temperament is a simple, simplistic and levelling approximation – nevertheless, the last two centuries of Western music wouldn't have been possible without it.

Equal temperament completely ignores both interval quality and interval difference. Its only concern is to close the circle of fifths, modifying each fifth by 1/12 of a Pythagorean comma (PC). This tuning system hides the general damage by distributing it evenly, as a relatively tolerable individual deficit. Perhaps it is not by accident that this equalitarian system was adopted during the second half of the 18th century, when the socially and philosophically dramatic break with the past of the French Revolution took place.

Dividing the Pythagorean comma into 12 equal parts will result in fifths slightly smaller than the pure fifth (700 cents instead of 701,955 cents), while the major thirds will be much larger than the pure third (400 cents instead of 386,3 cents). But how can we accept such dissonant thirds, why don't they bother us? The explanation could be that today's piano has much fewer harmonics than 16th- to 18th-century harpsichords and organs. Even Romantic organs have a rounder sound, poorer in harmonics but oriented toward the fundamental tones, unlike the highly individualised, very bright, even sharp, Baroque organ ranks. The dissonance of the thirds will therefore not be sensed

as annoying beats (for instance for A = 440 Hz we will have 13,86 Hz beats of the third F-A), but rather as a rolled consonant. Things become worse when organs have third ranks (Tierce, Sesquialtera, etc.), and their pure thirds will cause beats with the third of the major triad.

All major triads will be equally dissonant, with a deviation of 31,295 cents (the sum of the absolute deviation of the fifth, of the major and of the minor third), which is quite a significant amount. If for instance a (string) chamber music ensemble played such a chord in a cadence, it would sound so out of tune, that it would make us question the performers' professionalism. However, this system, which equalised and levelled all tonalities, was a necessary evil in the course of the history of music, and habit attenuated our wish to hear pure intervals. This is why to today's listener, a stranger to the Baroque sound-world, instruments tuned in irregular temperament would at first sound *out of tune*. It's a justified reaction, as the ear needs a time to readjust to an uneven soundscape (it's a phenomenon similar perhaps to what someone who has lived for a long time in artificial light and then suddenly goes out into the sunlight experiences – it takes a certain time to regain the ability of nuanced perception).

Even if this tuning was theoretically known as early as the 16th century, musicians showed no interest in it, believing it to lack expression. Only after about two centuries, when its features would match the 19th-century new views on music, would they begin, but not unanimously, to take it into consideration.

Pythagorean temperaments

Pythagoras starts with the natural (*pure*) fifth, 3/2 (*diapente*), the second musical proportion after the octave and the foundation of Pythagorean mystical philosophy (the sum of its terms is five and it also represents the interval formed with the fifth degree). Pythagoras' temperament is entirely fifth-oriented and disregards the thirds (the very opposite of the meantone system, entirely third-oriented). A concatenation of pure fifths starting with F leads us to the diatonic scale on which Greek modes are based: F-C-G-D-A-E-B, that is C, D, E, F, G, A, B. The tones have single value (major tone 9/8). The third, with two major tones (81/64 = 407,82 cents), very dissonant, larger than the equally temperate third, is not important for this music. Advancing toward the chromatic area, we can only arrive at a maximum of 11 from the 12 pure fifths, with a remaining fifth much smaller (by 1/2 PC), highly dissonant, and therefore not practicable. Such an interval with a great deviation from purity (no matter the direction) was of old called *wolf* (because it *howls* like a wolf) and can occupy various positions on the circle of fifths (G\$\$-Eb or B-F\$\$ or G-D etc.).

It's interesting to notice that the thirds which will contain this wolf (between the four fifths which make them up) will be very close to the pure third (384,36 cents as compared to 386,3 cents).

Adding that a further step was to give up the claim on 11 pure fifths and to distribute the wolf between two or three fifths, I will no longer insist on Pythagorean temperaments, as they are not the object of the period I am referring to. I should however emphasise that they have been used until the 15th century, when they were gradually replaced with meantone systems. Pythagorean systems were a perfect match for the musical aesthetics of an era when the fifth and the fourth were the consonant intervals par excellence and the third was dissonant (literally, not only figuratively). Intoning a clausula is enough to convince us how adequate to music this system is.

MEANTONE TUNING SYSTEMS

Pure (Praetorius) meantone tuning system

Historically, the emergence of meantone tuning systems marks a shift of interest, from the fifth to the third. The chord structures and the modulations created by the Renaissance polyphony require a re-evaluation of the need for pure intervals, and the third thus now becomes dominant. The keyboard instruments accompanying vocal ensembles or borrowing vocal compositions in tablatures must therefore satisfy the requirement that thirds be as close to the natural interval as possible, and in the largest number possible (of course, Pythagorean tuning systems didn't meet such requirements).

As shown, the succession of four fifths compose a third (plus two octaves). In Pythagorean tuning, four pure fifths generate, as we have seen, a much larger third than the pure interval (by 1 SC). To obtain a pure third we will have to *get rid* of the Syntonic comma, striking a compromise with one, two, three, or all four fifths. The more fifths we modify, the more tolerable this compromise will of course be. A minimal compromise is *narrowing* all four fifths by the same quantity – 1/4 SC. Four meantone fifths will be obtained, whose succession will generate a pure third. These fifths are smaller than the pure fifth by 1/4 SC:

- SC = 1,0125 [21,5 cents];

- Meantone fifth $5_{\rm M}$ = 1,495349 [696,578 cents] as compared to the pure (natural) fifth:

 $5_{\rm N} = 1,5$ [701,955 cents]).

With respect to subjective aural perception, because the deviation is relatively small compared to the pure fifth, this fifth (by itself) will not be perceived as out of tune, but likely as accompanied by slow beats in the middle register. Intoned together with the third, it will result, because of the third's purity, in a particularly clean chord.

This tuning is called *meantone* because it doesn't distinguish between the major and the minor tone. Indeed, if we take the meantone fifths generated by C, G, D, A and E we will obtain the pure C-E third, and the C-D and D-E tones will contain two meantone fifths each, so they will be identical, with a value between the major and the minor tone.

Meantone tone $T_M = 1,1180339$ as compared to:

- Major tone T = 1,125;

- Minor tone t = 1,111.

Basically, D will be exactly halfway between C and E.

A beautiful analogy with the layout of the keyboard instruments of the time can be made. The D key is in fact the axis of symmetry of the entire keyboard (as is G , the other axis which will be susceptible of analogy – see *Schlick temperament*, a further section of this study). Moreover, in many instruments, the D key was broader than the others. This was dictated not only by aesthetic and technical necessities, but by the same compromise which divided the octave in seven and in five equal parts, respectively (the white and the black keys) while preserving the symmetry of how the white keys are placed in relation to black keys. In practice, too, there was a marked preference for the tone D, as the D minor key was the most used then (a reminiscence of the most symmetric mode, the Dorian mode).

This is in fact the second compromise, now tied to the natural scale. It must be stressed that, despite its name, the division of the third into two equal tones or the unification of the minor and major tone (the geometric mean between the major and the minor tone) is just a consequence, and not a generating principle or a fundamental feature of this system. As a matter of fact, equal temperament, too, is a meantone temperament, as we only have one type of tones throughout the chromatic scale! And in this regard it is even more *meantone* than the meantone itself, which doesn't enjoy this particularity.

This is supported by the subjective perception of this meantone tone as well: the second degree deviates very slightly from the purity of the natural scale – a wholly insignificant deviation, since we are quite used to the equal temperament which completely standardized the small and subtle differences of the natural scale. For a very sensitive hearing, accustomed to historical tunings, this equality of the whole steps is perceived as a somewhat sorrowful, tense tone, because of the lowering of the second degree, especially in ascending conjunct motion (rather like in of Arab scales with an extremely lowered second degree). Let's not forget that this type of tuning was something of a novelty in its time, after a long period of just or Pythagorean intonation, and so the musical ethos couldn't overlook such deviations.

The wish for pure thirds now leads us to a third deadlock: with the requirement of enharmonic equivalency, three successive pure thirds generate an interval smaller than the octave. And this is the third compromise: one of the thirds will have to be sacrificed in order to preserve the purity of the other two. Thus modified, this third will be huge, larger than a pure third by precisely a Lesser diesis (LD = 41 cents) and extremely dissonant, larger even than a Pythagorean third (427,4 cents compared to $3_P = 407,8$ cents). We will arrive at a maximum total possible of eight pure thirds from the twelve. Obviously the other four highly dissonant thirds will limit the possibility to modulate to the tonalities of which they are constituent elements.

If we look at this tuning on the circle of fifths, we can see that it will not close unless we will sacrifice the twelfth fifth, which will be larger by 11/4 SC (the quantum by which the other eleven fifths were diminished) – a huge fifth, the wolf fifth (5_w), sounding more like a sixth. This fifth will be placed in the area of distant tonalities, according to the decision we make, to use a G[#] or an Ab: if we choose G[#] (as the pure third of E), we will place the fifth between G[#] and Eb, and if we choose Ab (with the Ab-C pure third) we will place the 5_w between C[#] and Ab (see Fig. 2 for the case of G[#]). And so it is that this problem-sound coincides with the keyboard's second axis of symmetry, the direct opposite of the D axis!



Fig. 2. Meantone temperament.

As we can see, the thirds containing the 5_w are sacrificed: B-Eb, F \sharp -Bb, C \sharp -F and G \sharp -C. The other eight pure thirds (marked with the dotted line) and their corresponding major chords are exceptionally pure, as the respective fifths' deviations of 1/4 SC are almost imperceptible within the chord.

This meantone temperament theoretically allows the approach of major tonalities up to four sharps and three flats, but once their dominants and subdominants are taken into account, only the major tonalities remain: C, G, D, A, and F, Bb. As concerns minor tonalities, the limitation is, because of their major dominants, stricter: only G, D, and A minor are practicable.

Minor thirds are particularity interesting: although not as relevant to this temperament as major thirds, here their deviation from the pure interval is identical to that of the fifths, namely 1/4 SC (nine out of the twelve). The other three, containing the 5_w , are very far away from purity and therefore not practicable.

It must be underlined that this tuning system doesn't aim to distinguish between tonalities, but to achieve the ideal of purity of as many tonalities as possible (i.e. of the major thirds). Indeed, the sound of meantone temperament is particularly consonant, bright and vigorous. The music of the time didn't see the limited modulations to those tonalities as a hindrance, and the pure sonority of the *Trias harmonica perfecta* justifies the dissemination and longevity of this system (in addition to its being easy to carry out).

The earliest description of the meantone temperament is that of Aron, at the beginning of the 16th century (Aron 1523), but we owe the fundamental source to Praetorius – which is why it was also known as Praetorian.

We must however insist on an aspect often overlooked, but maybe just as important as the study of the thirds – the sonority of the accidentals. We see that sharped notes are lower than flatted notes: Ab is for instance higher than G^{\sharp} , the difference being a LD (we consider Ab as the pure descending third of C and, of course, G^{\sharp} as the pure ascending third of E). There are two consequences of this.

First of all, this seems to contradict the principle (applied especially with regard to string instruments) according to which leading notes must be as high as possible, and as close as possible to the note to which they resolve. For today's musician, a halftone that large between the leading note and its resolution is unthinkable. How can this contradiction be resolved? We find the answer when we see that ideal sound changed over time. In that era, accidentals weren't perceived as leading notes, but as constituent tones in a certain harmonic context (for example as the third of the dominant chord), or

rather harmony was more important than melody. This way of hearing and performing music is tied not just to the meantone temperament: it remains valid throughout the late Baroque, in the world-sound of well temperament, until the second half of the 18th century (a proof thereof is Leopold Mozart's violin treatise, if we had to mention but one of the writings which addressed this issue; see Mozart 1756, 1985).

The second consequence is the quite distinctive sonority of the chromatic tones. Indeed, because halftones are irregular, the themes from chromatic fantasies or toccatas, the dissonant leaps and the false relations will create a special aural effect. It is perhaps one of the reasons why composers of the time showed a predilection for such rhetorical figures as Passus duriusculus, Saltus duriusculus and for Relatio non harmonica, which generate an acoustic conflict, a real and not just an abstract sonic tension. There was another quite important field where meantone halftones intervene - ornamentation. For example, a trill on F[#] will call attention to the *major* halftone F[#]-G. It might appear as a case of inaccurate tuning, but in fact this expansion, this dilatation of the trill's interval is intentional; many wind instruments treatises provide tables with modified fingering charts to help with ornamentation, that is, to modify intonation so that the interval of the trill is larger than the interval that would normally be intoned (in conjunct motion for instance). This way, the ornament is highlighted, in conformity with the aesthetic of the Baroque.

But still: besides the intentionally out of tune sounds, how can we *hide* the limitations imposed by this tuning system? For harpsichordists, things are relatively easy: they must decide whether a certain work calls for a G^{\sharp} or for an Ab, group the pieces accordingly, and retune all G^{\sharp} s in the break between the two sets (in the early Baroque we rarely come across both a G^{\sharp} and an Ab in the same score). The literature mentions another way of *mask-ing* wrong notes – ornamenting them, thus concealing the sensation of an out of tune sound.

In order to be able to play works in tonalities beyond the Praetorian meantone limitations, organists rather frequently resorted to transposition. A number of works reached us both as originals and as transpositions, including some by Bach (the Toccata BWV 566 in E major has a version in C major, and Bach's early works were for that matter written for meantone organs). For reasons easy to understand, the organ is, temperament-wise, the most *conservative* instrument. In the important musical centres, organs started undergoing changes in the meantone temperament in the second half of the 17th century. Elsewhere, these changes were delayed due to the conservative

spirit or the lack of financial resources, so that original meantone systems fortunately survive to today.⁴

It's obvious that, with the emancipation of the early Baroque instrumental music, Praetorian meantone limitations had to be left behind. And there were two ways of saving the meantone spirit and principle: by taking the absolute approach, or by striking a compromise.

The chromatic harpsichord

The desire to modulate and transpose (a frequent practice of the time) called for the extension of the number of tonalities in which the instrument could play. Further along the circle of fifths, without feeling constrained to close it, we will continue to concatenate meantone fifths, obtaining new pure thirds and transforming the circle into a spiral (see Fig. 2). This spiral can be prolonged towards the double sharps and double flats (G \sharp , D \sharp , A \sharp , E \sharp , B \sharp , F_x... and Ab, Db, Gb, Cb, Fb, Bb...).

This was technically achievable with the building of instruments with split chromatic keys. The most useful accidentals are the pairs $G^{\sharp}-Ab$ and $D^{\sharp}-Eb$, and the instruments possessing these split keys were quite frequent at that time (we still have harpsichords and even organs with 14 keys per octave, that is, with two split chromatic keys). This type of instrument has of course its limitations: eight possible major tonalities, from Eb major to E major, and five minor tonalities, from C minor to E minor.

For the followers of non-enharmonic equivalency, this extended spiral opened up new possibilities. They viewed enharmonic equivalency as a crude compromise, and they were perfectly right, because enharmonizing brought about the loss of a whole world, that of modulation subtlety (enharmonic modulation in particular). This meant giving up a dimension, and this meant compromising, but the whole history of music from the 18th century to the present day would not have been conceivable without it (see Fig. 3).

New instruments thus appeared, with more than two split chromatic keys, and with up to 17, 19 or even more sounds per octave (the harpsichord described by Nicola Vicentino in 1555 and built by Vito Trasuntino in 1606 has 31 keys per octave and is held in the Archaeological Museum of Bologna; see Vicentino 1555). Of course, as the number of keys grew, so

⁴ The oldest organ in Romania, built by an anonymous maker in 1699, is in Rupea (Brasov County). The original 1/5-comma meantone temperament was redone on the occasion of the instrument's restoration in 2012. This is the only meantone instrument in Romania.

did the technical problems posed to both makers and performers. The time's instrument of choice had 19 keys per octave and is described by Praetorius in *De Organographia* as "Cimbalo Universale – instrumentum perfectum si non perfectissimum" (Praetorius 1619: 65). In addition to its five split chromatic keys, it featured another two small chromatic keys placed between B-C and E-F, corresponding to the sounds Cb and E\$, respectively. The distribution of the split keys results in a rather complex, relatively comfortable, with a performer-friendly logic keyboard.



Fig. 3. The spiral of fifths.

This chromatic harpsichord allows playing in the major tonalities of G_b to F_a^{\sharp} and in the minor tonalities of E_b minor to F_a^{\sharp} minor. There are five reconstructed such universal harpsichords, one of them at the Organeum in Weener, Germany. From personal experience I can say that tuning it isn't by far as complicated as it might seem, and the technical difficulties of playing on a 19-key keyboard are fully rewarded by the refined sonorities of the modulations which this instrument, rightly considered perfect, allows for.

1/5-comma meantone temperament

The meantone temperaments described below go in the direction of a compromise designed to ameliorate the limitations imposed by the pure meantone temperament. Still oriented as they are towards the same spirit, of a third as pure as possible, they show less insistence on absolute purity.

If the Syntonic comma is distributed between five instead of four fifths as in the pure meantone temperament, the fifths will be narrowed by only 1/5 instead of a 1/4 SC and will deviate less from the pure interval. The thirds will not be purer, but larger by 1/5 than the pure interval (they will have the same deviation as the fifth). The $5_{\rm W}$ will still be there, just somewhat smaller than

in Praetorian meantone (24 cents instead of 35 cents), and the major triad loses some of its purity (17 cents instead of 10 cents). In fact, nothing much is gained with the loss of the thirds' absolute purity – on the contrary, this tuning system is rather more difficult to perform, because of the impossibility to control the fifths by verifying the purity of the thirds.

1/6-comma meantone temperament

In the attempt to attenuate the fifth's *narrowness*, various divisions of the Syntonic comma were experimented. One with 1/6 is mentioned by Sorge in relation with Gottfried Silbermann and is worth taking a look at (see Sorge 1748). The main disadvantage, despite an amelioration of the fifths, is, again, the 5_W , insufficiently attenuated. As regards the thirds and the fifths, this tuning system is approximatively halfway between the pure meantone and the equal temperament. The evenly distributed 1/6 of a comma (both SC and PC) marks the limit of irregular temperaments, as fractions smaller than 1/6 will generate tuning systems already corresponding to the uniformity of the equal temperament. It's also interesting to make a comparison with the tuning system which uses 1/6 PC evenly distributed between six fifths described by Vallotti. Neither the 1/5 SC system nor this one creates differences between tonalities, because the fraction of the comma is evenly distributed.

The presence of the 5_w does bother here, especially because Eb, much too high, can't be accepted as D\$. To remove this inconvenient, Werckmeister recommends either adopting his system or splitting this key (see Werckmeister 1691). Another possible solution would be lowering Eb to a pitch acceptable as D\$, which, theoretically, corresponds to the splitting of the 5_w between two fifths, G\$-Eb and Eb-Bb.

That Silbermann continued to use this tuning system (Sorge criticized him for his conservatism; see Sorge 1748) instead of one well-tempered shows that starting with the second half of the 17th century and until around mid-18th century different tuning systems coexisted. Sources of the time indicate that, geographically speaking, in France, England and Italy the principle of meantone temperament was maintained through the various ways of unevenly distributing the fractions of the SC, while in Germany and other European countries musicians and theoreticians turned toward the diversity of the well-tempered systems. Organ tuning also changed slower than that of harpsichords or clavichords, always up-to-date with the specific musical requirements. With the uneven distribution of the fractions of the comma, richer modulations became possible. The many ways of distributing these fractions produced a variety of meantone temperaments known, because they were mentioned in a great number of French sources, as "French tuning systems".

Mersenne temperament

Like the pure meantone, the temperament described by Mersenne is based on the quarter of a Syntonic comma, while in order to diminish the 5_w it distributes it between three fifths (Mersenne 1636, 1649). On the circle of fifths, we concatenate meantone fifths (diminished by 1/4 SC) on the diatonic sounds (F, C, G . . .), continuing up to G[#], and then, going down from F, two rough fifths (large, larger than a pure fifth). Mersenne's description is rather imprecise in that it doesn't say how large these fifths must be compared to the remaining G[#]-E^b fifth. We can consider, for the sake of symmetry, that the two fifths have the same deviation as the meantone fifths, but in the opposite direction (they will thus be augmented by 1/4 SC). In this version, compared to the pure meantone, three extra tonalities will be possible, E major, E minor and G[#] minor, but the flat tonalities will no longer be as pure (because of the two impure thirds, Eb-G and Bb-C). The preference for sharp tonalities is constant in the French music of the time, and, as a result, they are a favourite of "French tuning systems". The lingering presence of the $5_{\rm W}$, albeit attenuated, still limits modulation. As a novelty, we can see a slight outlining of tonalities: there are no longer pure tonalities and not practicable tonalities, but starting from F major towards E major we have pure tonalities, then the triads gradually *worsen* to C[#] major and G[#] major, then they progressively improve towards F major.

Rameau temperament

This French tuning system, very frequent in the 18th century, appears in many sources (including Rameau, from whom it takes its name; see Rameau 1726, 1737) and it is the first to propose the removal of the 5_W by distributing it between several fifths in the chromatic area of the circle. Rousseau's description of this temperament, while the clearest and most complete, is nevertheless somewhat imprecise, leaving room for different interpretations on its construction. This is why I considered it important to reproduce the original text from Rousseau's dictionary (see Fig. 4).

The essence of this system is the building of pure thirds on the most important tonalities, C major and G major, first by concatenating meantone fifths and then, from F \sharp onwards, ever larger fifths to G \sharp . This progressive expansion is not quantised in any way, and it can be only verified by ensur-

ing that the E-G[#] third is good or at least acceptable (we exclude the pure version, as it would take us back to Mersenne). We must solve another of Rousseau's inconsistencies: the descent from C in fifths, at first minor, then gradually larger, comes in contradiction with the character of the tonalities of Eb major and Bb major, dark and somewhat rough because we don't dispose of enough fifths to grade their augmentation. One last inadvertency, which we can attribute to the confusion between Eb (D#) and Db (C#), would be that Db taken as C# must form a fifth in tune with G[#].

502

ΤΕ M.

Mais il faut que le mème Son mi, qui fait la Quinte de la, ferve encore à faire la Tiesce majeure d'ut; il faut que le même fi Dièle, qui forme la douzième Quinte de ce même ut, en fasse aussi l'Octave, & il faut enfin que ces différens Accords concourent à constituer le fystème général sans multiplier les Cordes. Voilà ce qui s'exécute au moyen du Tempérament.

Pour cela 1º. on commence par l'ut du milieu du Clavier, & l'on affoiblit les quatre premières Quintes en montant, julqu'à ce que la quatrième mi fasse la Tierce majeure bien juste avec le premier Son ut; ce qu'on appelle la première preuve. 2º. En continuant d'accorder par Quintes, dès qu'on est arrivé fur les Dièfes, on renforce un peu les Quintes, quoique les Tierces en souffrent, & quand on est arrivé au fol Dièle, on s'arrête. Ce fol Dièle doit faire, avec le mi, une Tierce majeure jufte ou du moins souffrable; c'est la seconde preuve. 3 °. On reprend l'ur & l'on accorde les Quintes au grave; favoir; fa, fi Bémol, &c. foibles d'abord ; puis les renforçant par Degrés, c'eft-àdire, affoiblissant les Sons julqu'à ce qu'on soit parvenu au re Bémol, leguel pris comme ut Dièle, doit se trouver d'accord & faire Quinte avec le sol Dièfe, auquel on s'étoit ci-devant arrêté; c'est la troisième preuve. Les dernières Quintes se trouveront un peu fortes, de même que les Tierces majeures ; c'eft ce qui rend les Tons majeurs de fi Bémol & de mi Bémol fombres & même un peu durs. Mais cette dureté fera supportable fi la Partition est bien faite, & d'ailleurs ces Tierces, par leur fituation, font moins employées que les premières, & ne doivent l'être que par choix.

Les Organistes & les Facteurs regardent ce Tempérament comme le plus parfait que l'on puisse employer. En effet, les Tons naturels jouissent par cette méthode de toute la pureté de l'Harmonie, & les Tons tranfpolés, qui forment des modulations moins fréquentes, offrent de grandes reflources au Musicien quand il a besoin d'expressions plus marquées : car il est bon d'observer, dit M. Rameau, que nous recevons des impressions différentes des Intervalles à proportion de leurs différentes altérations. Par exemple, la Tierce majeure, qui nous excite naturellement à la joie, nous imprime jusqu'à des idées de fureur quand elle

Fig. 4. Rousseau's description of Rameau's temperament (Rousseau 1768: 502).

Many theoreticians today proposed a number of techniques of reconstructing this tuning system. Based on personal interpretation and the experience I gained while practicing tuning, I chose to analyse three versions.

Rousseau's description leaves much room for subjective interpretation, which is why we must stick to the essentials and follow several principles specific to French music of the time:

1. First of all, this meantone temperament allows pure thirds only with regard to closely related tonalities, in order to remove the 5_w and to allow modulation to all tonalities;

2. French music favoured sharp tonalities to the detriment of flat tonalities;

3. Rousseau next shows why this tuning system is considered by the musicians of the time as the *most perfect* (*sic*) ever: closely related tonalities enjoy all of the harmonic purity and distant tonalities provide, through modulation, rich expressive resources. Of course, this has to do with the Baroque aesthetic ideal of nonuniformity and differentiating tonality colour by treating constituent intervals as not equally consonant (according to the doctrine of the affections – *affetti* – applied to tonalities).

In reconstructing this tuning, I chose a two-step gradation in the flats area, from the C-G meantone fifth to the F-C pure fifth and from the F-C fifth to the Bb-F fifth (larger than the pure). I also placed the most impure fifths between Eb-Bb and Bb-F and I maintained the same degree of impurity, because a gradation of these tonalities (Eb major, Bb major and F major), caused by the different fifths which the respective thirds contain (Eb-G, Bb-D and F-A), will nonetheless take place.

Likewise, in the sharps area I explored three possibilities of a graded augmentation of the fifths, starting with B-F[#]: one three-step, one two-step, and one featuring a sudden transition. I obtained the values of these steps by dividing the difference from the meantone to the pure fifth (approx. 5,4 cents) by three (approx. 1,8 cents) and by two (approx. 2,7 cents), the sudden transition having as much as 5,4 cents.

Some evaluations of these versions are necessary: they all respect the condition of the purity of the C-E and G-B thirds, and the cleanest chords are F, C, and G major; all favour sharp tonalities (compare D major with Bb major, A major with Eb major and then the deviation of Eb major with the similar deviation in the sharps area). We notice a beautiful gradation of sharp tonalities in the three-step version, but it is the one which has the most dissonant chords in the F\$-Eb major area. The version with no intermediate steps is more abruptly graded and features therefore a more sudden worsening of the sharp tonalities, but instead the distant tonalities (F[#]-E^b major) have the least dissonant chords of all three versions (see Fig. 5).



Fig. 5. Rameau's temperament (no intermediate steps).

There is a major advantage of the sudden transition version that I want to mention: the tuning can be performed very easily, safely, and quickly. This is not to be ignored, as the first function of tuning systems was one practical, and the theoretical arguments played but a supporting role. Indeed, we only have pure test intervals: C-E and G-B as pure thirds, then pure fifths from B to Eb, then, downward, C-F. The most dissonant fifths originate in the even distribution of the others between Eb-Bb and Bb-F (that is, Bb is adjusted until the two fifths sound just as bad).

Of course, for the ear of today's musician the degree of dissonance of distant tonalities is still difficult to accept, but Rameau temperament is the best fit for French music of the 18th century, highlighting, during modulations, the distinct affections of the various tonalities. This temperament can be considered the most irregular and differentiated among meantone systems and it shows the many possibilities of constructing a meantone tuning. We can accept the hypothesis that each musician had their own way of tuning their instrument, but what was decisive was the measure in which that temperament served the music (there are testimonies of tuning competitions which evaluated both speed and accuracy).

The desire to have more acceptable fifths and less dissonant distant tonalities manifest in meantone temperaments which divide the Syntonic comma in five equal parts, distributed between the fifths of the diatonic area, while the next fifths from the chromatic area are gradually augmented. 1/5 comma tunings can be performed in a variety of ways, depending on how the fraction is distributed and how the remainder is hidden in the area of distant tonalities (which will be improved to the detriment of the purity of close tonalities – none will any longer have a pure third in the tonic chord). At the same time, the differentiation of tonalities will be subtler, and consequently the modulations less abrupt. It's interesting to notice that these French tuning systems, even if they start from a completely different principle (meantone, related to the SC) than the well-tempered systems (which start with the division of the PC), eventually reach the same result – the ideal sound of the late Baroque: a differentiation of affections of tonalities generated by the different qualities of the constituent intervals. Before this chapter is over, I will present a rather peculiar case in the mass of historical tuning systems.

Schlick temperament

Schlick's tuning system is the earliest description of a nonuniform tuning (see Schlick 1511). We can't say for certain that it is a meantone tuning, but references to thirds makes this classification plausible. Besides, it anticipates by its characteristics the nonuniform (of the Rameau type) and well-tempered tuning systems by at least two centuries, so that it could very well be labelled as belonging to the latter. Although it appears as a superb theoretical construction, it is designed to be a practical description to help organ builders. An unexpected particularity for its time is the fact that none of the intervals is pure, and consequently this tuning is difficult to perform. All thirds will be larger than the pure third, but C-E, G-B and F-A will be better than the others; the fifths from the diatonic area will be narrowed so as to generate those better thirds. The B-F \ddagger , F \ddagger -C \ddagger fifths and the descending C-F and F-Bb fifths (marked 5+) are obtained by tuning G \ddagger as a compromise with Ab, that is, the E-G \ddagger and Ab-C thirds will be equally dissonant (see Fig. 6).

This tuning is remarkably symmetric. As the quantity by which the fifths are diminished is not specified, we will take into consideration four cases of six diatonic fifths (1/5 SC, 1/6 SC, 1/5 PC, 1/6 PC). For the sake of symmetry, we will assume that the C#-G# and G#-Eb fifths have the same deviation as the six, but in the opposite direction (they are augmented by the same quantity). We can thus comparatively admire the symmetry of tonalities with the symmetry of the circle of fifths.

There is no proof of this temperament being used at the time of its invention, and there are no other later references to it. Obviously Schlick was ahead of his era by approximatively two centuries, or rather, his thinking didn't match the contemporary musical requirements. Only in the late Baroque could this vision, differentiating tonalities in such a refined manner, have found its place. Such *accidents* make it clear that we cannot talk about a good or bad tuning system as such, but about how much one such temperament fits and is representative for the aesthetic ideal of its day.



Fig. 6. Schlick's temperament.

A similar case is Vicenzo Galilei's treatise, which describes a tuning system quite similar to the equal-tempered, but based on rational numbers. A ratio of 18:17 is indicated, which means 99 cents, so that it is a good approximation of the half step of equal temperament (Galilei 1581). Salinas too describes, in 1577, equal-tempered tuning (Salinas 1577). Nevertheless, the entire literature of the following two centuries doesn't take equal-tempered tuning into consideration, because its uniformity didn't correspond to the musical conception of the day.

WELL-TEMPERED TUNING SYSTEMS Werckmeister temperament

Around the end of the 17th century, meantone temperaments were being abandoned, as no longer suitable to the musical mindset and aesthetics of the era. The division of the Syntonic comma had the disadvantage of forming a remainder (by narrowing the diatonic fifths), which then had to be attributed to some chromatic fifths in their turn becoming wolf fifths (even if such fifths were still accepted by some meantone temperaments). The meantone principle had to be abandoned because it couldn't evolve (this is why it's incorrect to say that meantone temperaments evolved toward equal-tempered tunings). The new requirements were:

1. The possibility to modulate to all tonalities, that is, the complete lack of wolf intervals, and closing of the circle of fifths;

2. A differentiation as refined as possible of the expressivity of each tonality, with a slight preference for the closely remoted.

The first condition requires that fifths have a value at most equal to the natural fifth. To close the circle, several of these fifths must be narrower. And so the PC returns as the measure of well-tempered tunings. The differentiation of tonalities depends on the ingenuity with which these divisions of the Pythagorean comma are distributed.

Werckmeister describes three tuning systems based on the division of the PC, among which only the first is worth considering (the other two are particularly precarious, so it is no wonder that Werckmeister himself declared that he preferred the first). The PC is divided into four equal parts distributed between C-G, G-D, D-A and B-F[#] (see Fig. 7).



Fig. 7. Werckmeister's temperament.

The other fifths are pure, so that the tuning is simple both in theory and in practice. But it has neither the beauty of the symmetry of Schlick's temperament, nor its efficiency as concerns the sinuous profile of the tonalities. An interesting particularity is the isolated position of the fourth narrowed fifth (B-F\$), which ameliorates the tonalities of D major and A major, as compared to when the four narrowed fifths would have been placed in a row between C and E. The F-A and C-E thirds will be the closest to purity, and F\$#-Bb, C\$#-F and G\$#-C (the so-called Pythagorean thirds), the most dissonant. Unfortunately, the fifths narrowed by 1/4 PC are rather dissonant (slightly more dissonant than meantone fifths) and are part of the most frequently used tonalities.

Vallotti temperament

The Vallotti temperament, described by Tartini, divides the PC in six equal parts, distributed between the fifths formed by the diatonic tonalities (Tartini 1754). The other fifths (from the chromatic area) are pure. We notice the symmetry of the circle of fifths (see Fig. 8), which occasions a comparison with Schlick's temperament.



Fig. 8. Vallotti's temperament.

The most consonant thirds will be F-A, C-E and G-B (marked with the dotted line) and the most dissonant, the Pythagorean thirds B-Eb, F \sharp -Bb and C \sharp -F. The axis of symmetry still goes through D and G \sharp , the diatonic tonalities will contain temperate fifths, and the chromatic ones, pure fifths. What is very interesting is that there are only two types of fifths and that their values are symmetrically placed around the fifth from the equal-tempered tuning (0 and 1/6 as compared to 1/12). Compared to Schlick's, this tuning system generates a less sinuous and graded profile of tonalities, because of the equal division of the comma and the uniform, compact distribution of the fractions of the comma. We nevertheless see that the most distant thirds are more tolerable than with Schlick. We can say that the Vallotti temperament is the last well-tempered tuning, both chronologically and as regards the efficiency of a refined differentiation of tonalities. It is however ideal for early Classicism galant style.

Between these two extremes – Werckmeister and Vallotti – there were numerous well-tempered tuning systems. The different ways of dividing the PC (equal or unequal, from 1/4 to 1/6 and even going as far as 1/7 and 1/8) as well as the multitude of possibilities of distributing these fractions generated, over the 18th century, the subtlest ways of differentiating tonalities. To this testify the many historical sources featuring descriptions of the specific character of tonalities.⁵

One of the most refined tuning systems in the 18th century Germanspeaking countries is the Neidhardt temperament.

Neidhardt temperament

Among the systems described by Neidhardt, one deserves special attention due to its heterogeneous and asymmetrical aspect, which breeds an extremely interesting configuration of tonalities (see Neidhardt 1732). Fifths are tempered by 1/6 PC and 1/12 PC and their distribution is both in the diatonic and in the chromatic area. The other fifths will be pure (see Fig. 9).



Fig. 9. Neidhardt's temperament.

We notice that it anticipates Vallotti's system (by tempering with a diatonically distributed 1/6 PC) and equal-tempered tuning (by the presence of the 1/12 PC fifths) as well as the hiatus E-B, inspired perhaps by Werckmeister.

The configuration of tonalities exhibits refined differentiations and an amelioration of the distant tonalities (the most dissonant is F^{\sharp} major).

⁵ Tonartencharacteristik (non-exhaustive list): Rousseau (1691), Charpentier (1692), Masson (1697), Mattheson (1713), Rameau (1722), Heinichen (1728), Mizler (1736-38), Mattheson (1739), Stössel (1737), Quantz (1752), Marpurg (1776), Junker (1777), Vogler (1779), Ribock (1783), Schubart (1784/85), Cramer (1786), Grétry (1797), Vogler (1798), Knecht (1803), Koch (1807), Hoffmann (1815), Rochlitz (1824), Schumann (1835), Hand (1837), Berlioz (1856), Auhagen (1983).

"Well-invented" temperaments

Overall, as concerns the many tuning systems of the 18th century, there is the issue of relating them to the emblematic *well temperament work* – Bach's *Well-Tempered Clavier (WTC)*. Interest in tonal exploration is manifest as early as the second half of the 17th century (Froberger – *Canzone durch alle 12 Tonarten*, unfortunately lost) and culminates with the beginning of the 18th century (J. C. F. Fischer, J. Mattheson, etc., then J. D. Heinichen, G. A. Sorge, J. Ph. Kirnberger, etc.), *WTC* being neither the first nor the only work in this genre. Such *circular* works mark the definitive break up with meantone systems and affirm the aesthetics of hierarchy in regard to differentiating the affections of tonalities.

Many musicians – musicologists or performers – attempted, in the last 30 years, to determine to what extent we can talk, with respect to *WTC*, of a tuning system devised by Bach himself, of one ideal, or of one which would at properly highlight the different character of each tonality, synthesising new tuning systems in the spirit of those well-tempered.

Barnes had the patience and tenacity to analyse the major thirds in *WTC* and to sort them by the degree of perceptibility in five categories, from very obvious to imperceptible. By mathematical operations, he obtained a *hierarchisation* of the major thirds and synthesised a matching tuning system (Barnes 1979).

F♯	C#	Ab	Eb	Bb	F	С	G	D	A	Е	В
32	41	74	72	99	124	139	68	48	73	46	41

Table 1. Barnes' hierarchisation of major thirds.

The diagrams he made both for his own tuning system and for other historical well-tempered tunings show how well they fit *WTC*. It's interesting to mention that Barnes' is preferable to other systems (such as Werckmeister III), but it is surpassed by Neidhardt's from 1729. The result of an analysis, it is therefore highly objective. As for the analysis itself, it too is correct, as it is historically certified by its correlation to Neidhardt's tuning even more than to Barnes' own (see Fig. 10).

Kellner starts from another extremely important factor in aural perception – the beats phenomena. As the well-tempered tuning systems imply a differentiated deviation from purity and at the same time a *negotiation* between fifths and thirds, he demands that the C₁-G₁ fifth have the same number of beats as the C₁-E₁ third, to *beat* together (the fifth beats by diminution, the

third, by augmentation). From this premise, by calculations, the narrowing (the deviation from purity) of the fifths C-G-D-A-E is obtained. We notice that the result is extremely similar to the case of dividing the comma in five (1/5). The difference is almost imperceptible:

- The Kellner fifth = 697,2784 cent;
- The 1/5 fifth = 697,2630 cent.



Fig. 10. Barnes' temperament.

To close the circle of fifths, there is a remainder (it, too, close to 1/5 of a comma) which Kellner places on the B-F[#] fifth. We notice the same *trick*, moving the last narrowed fifth somewhat away from the block of four by the E-B hiatus (see Fig. 11). By doing so, he avoids the sudden alteration of tonalities which modulation toward the sharps engenders, and the transition to the more distant tonalities is softened. This characteristic is actually found in many of the historical temperaments (Werckmeister III, Neidhardt etc.), but also in Barnes', where it was the outcome of an objective analysis.

Kellner claimed that this temperament is that of Bach himself and that he discovered it by esoteric research, resorting to numerology and deciphering Bach's seal (Kellner 1976). These statements, neither historically nor analytically proven, don't make this temperament less significant, of course. The premise from which he starts – that of beats – is both unifying and just as meaningful with regard to aural perception and with physical (acoustic) quantification. Besides, 1/5 PC is a sort of border area of the division of the comma: 1/4 sends us to the meantone system (as it is close to 1/4 SC) and 1/6 makes differentiating the tonalities less dramatic. It is a sort of "goldener Mittelweg".



Fig. 11. Kellner's temperament.

Lastly, Lehman had the inspiration to study carefully *WTC*'s well-known title page (see Fig. 12). Above the title is what appears to be an ornament drawn without too much care for the aesthetic. A closer examination reveals that not all loops are identical, and if we look at it with Lehman's eyes we can even notice that there is a small C above the C in "Clavier" – thus, the sound C is noted – and that there are 11 loops. These loops are drawn in a variety of ways: some simple, some with an extra, small loop, some more complicated, with two extra, small loops. The sequence of the fifths and the details of this drawing are much more clearly discernible if we put it upside down. Starting from these graphic considerations, Lehman synthesises a tuning system which may very well be the one Bach intended for *WTC* (Lehman 2005). But instead of speculating, it's more profitable to analyse this temperament.



Fig. 12. *Title page of Bach's* Well-Tempered Clavier.

This graphic ornament doesn't of course offer quantitative information. We find the narrowed fifths on the diatonic tonalities, but starting with F (as in Vallotti, Barnes etc.). It's almost obvious that we are talking about 1/6 of a comma, it's just that we have only five complicated loops. The author sees in the three simpler loops a narrowing by 1/12 of a comma, which is placed in each of the fifths C \sharp -G \sharp -Eb-Bb. The closing of the circle of fifths is thus exceeded by 1/12 of a comma, compensated by the Bb-F fifth, which will be augmented. The number 3, at the left end of the drawing, is interpreted as the number of beats of the F-A third (in the tenor register).



Fig. 13. Lehman's temperament.

There are common features with the Neidhardt and the Vallotti temperaments among others, but the previously-mentioned fifth disparity is missing (see Fig. 13). This will prevent the amelioration of the A-C[#] third which results from Barnes' analysis and from other historical tunings. A very interesting particularity would be the similarity to Rameau's temperament as concerns the irregularity generated by the augmentation of the fifth in the flats area. What in the Rameau temperament meant improving the sharp tonalities at the expense of the flat tonalities – typical for the French music of the time – here turns into putting at disadvantage (albeit slightly, since it concerns only 1/12 of a comma) the flat tonalities, thus it even opposes the results of Barnes' analysis. Of course, the iconographic proof is consequential in appreciating this tuning system strictly from the WTC perspective. Thus, it is another valid tuning from the multitude of well-tempered tuning systems.

Sources of the time testify that Bach voiced and tuned his harpsichord himself. As well, all major thirds were somewhat larger (*scharf*), which clearly

shows the difference from meantone systems. We will probably never know how Bach tuned his harpsichord and if he had a preferred temperament or whether he used various systems according to the character and tonal plane of the respective work. Today, musicians specialising in Baroque music adopt and adapt various systems according to the tonal plane of the works they play, which seems to be the best approach. Only an unbiased exploration of the vast universe of temperaments will bring new sonorities and new colours of the various modulations to light, enriching the expression and the meanings of this music.

English version by Maria Monica Bojin

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